Geo-Spatial Reasoning: Using RCC in the Semantic Web

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Bridge between GIS and OWL

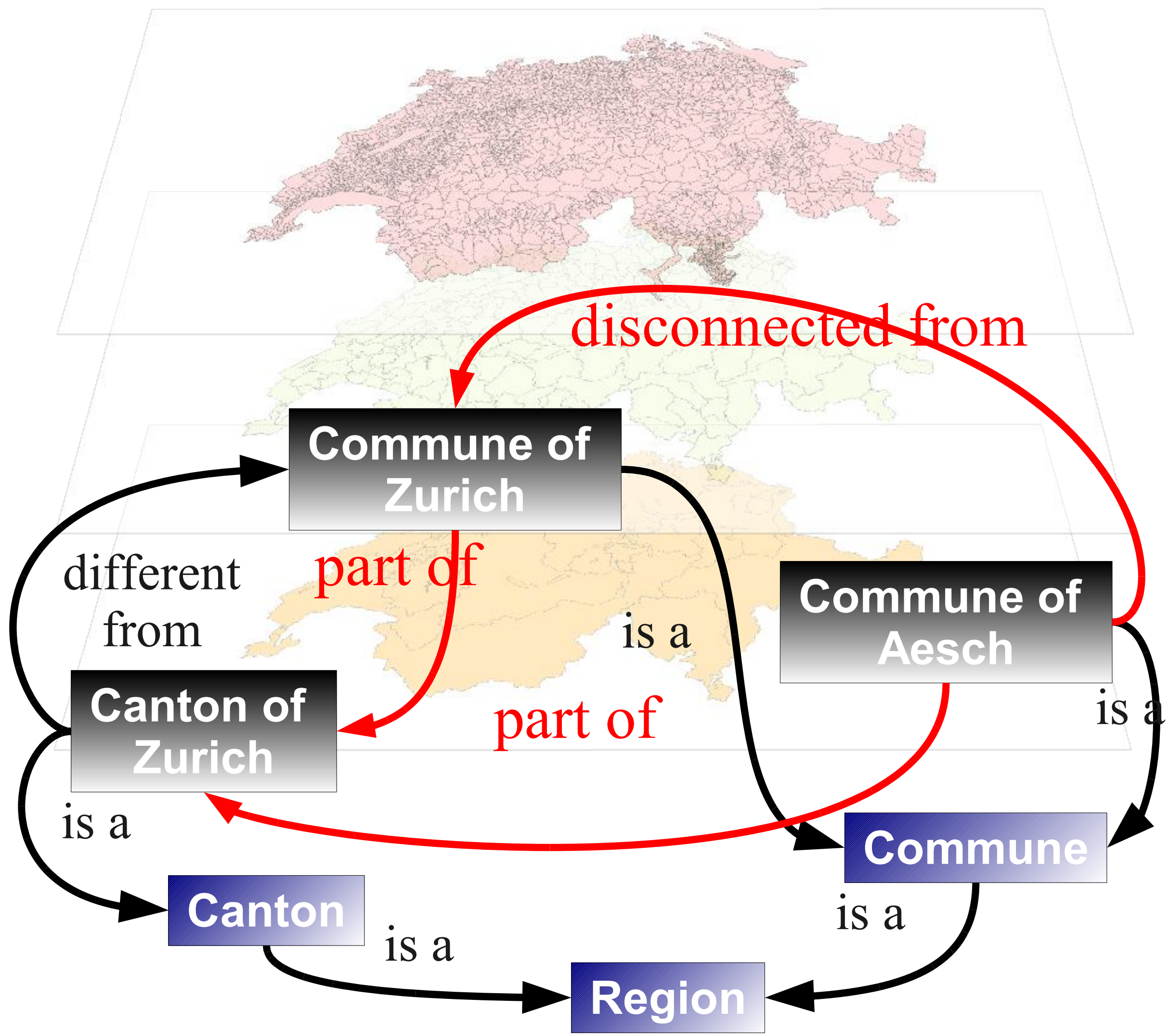
- GIS layers
  - Geometry
  - Topology

QUANTITATIVE

- **RCC**

- Domain Ontology
  - Concepts
  - Relations
  - Rules

QUALITATIVE



Region Connection Calculus (RCC)

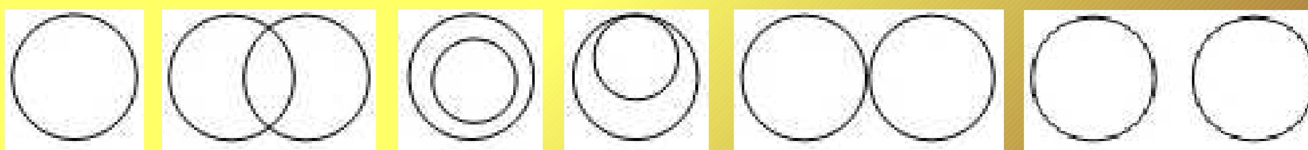
- integrate **topological**, and **conceptual** information
- describe **relations between regions**
- regions support **either spatial**
  or temporal interpretation

Randell et al. (1992)
Bennet (2000)

RCC-1

Spatially Related

$SR(x, y) \equiv_{def} \top(x, y)$

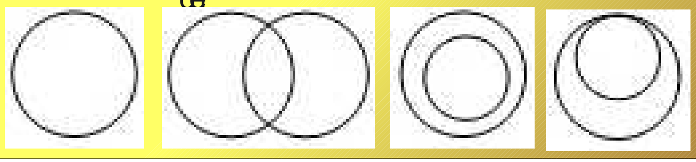


- One basic primitive dyadic relation:
  - ConnectsWith(x,y)
- One basic auxiliary relation:
  - PartOf(x,y): $P(x, y) \equiv_{def} \forall z[C(z, x) \rightarrow C(z, y)]$

RCC-2

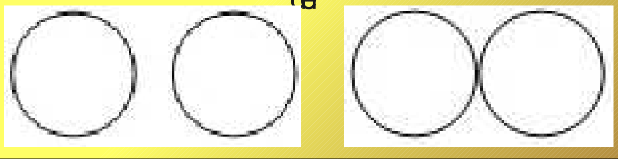
Overlaps

$O(x, y) \equiv_{def} \exists z[\, P(z, x) \wedge P(z, y)]$



DiscRete from

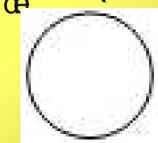
$DR(x, y) \equiv_{def} \neg O(x, y)$



RCC-3

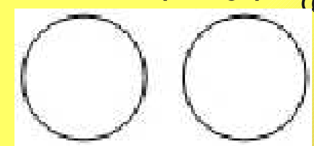
EQual to

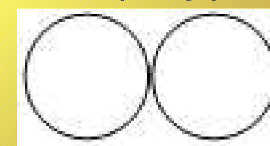
$EQ(x, y) \equiv_{def} P(x, y) \wedge P(y, x)$



DiscRete from

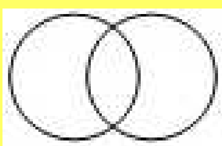
$DR(x, y) \equiv_{def} \neg O(x, y)$

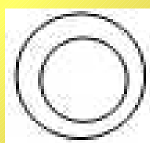


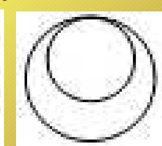


Overlaps Not Equal

$ONE(x, y) \equiv def \; O(x, y) \wedge \neg EQ(x, y)$



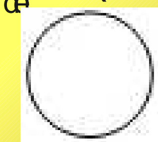




RCC-5

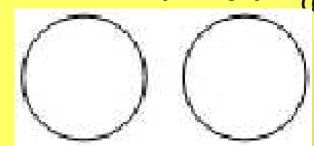
EQual to

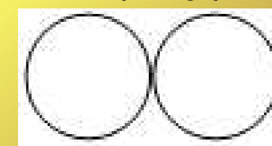
$EQ(x, y) \equiv_{def} P(x, y) \wedge P(y, x)$



DiscRete from

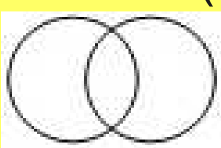
$DR(x, y) \equiv_{def} \neg O(x, y)$





Partially Overlaps

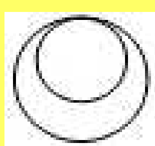
$PO(x, y) \equiv_{def} O(x, y) \wedge$

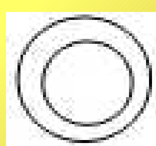


$\neg P(x, y) \wedge \neg P(y, x)$

Proper Part of

$PP(x, y) \equiv def\ P(x, y) \wedge \neg P(y, x)$





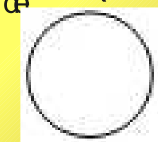
Inv. Proper Part of

$PP(x, y) \equiv def\ P(x, y) \wedge \neg P(y, x)$

RCC-8

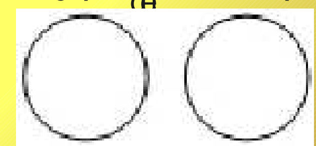
EQual to

$EQ(x, y) \equiv_{def} P(x, y) \wedge P(y, x)$



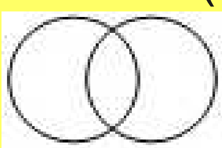
DisConnected from

$DC(x, y) \equiv_{def} \neg C(x, y)$



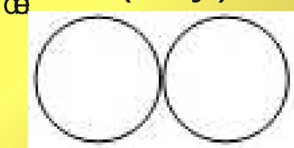
Partially Overlaps

$PO(x, y) \equiv_{def} O(x, y) \wedge \neg P(x, y) \wedge \neg P(y, x)$



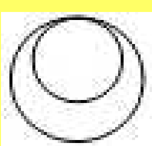
Externally Connected to

$EC(x, y) \equiv_{def} C(x, y) \wedge \neg O(x, y)$



Non Tangential Proper Part of

$NTPP(x, y) \equiv_{def} \neg PP(x, y) \wedge \exists z[EC(z, x) \wedge EC(z, y)]$

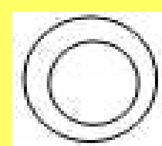


Inv. Non Tangential Proper Part of

$NTPPi(x, y) \equiv_{def} NTPP(y, x)$

Tangential Proper Part of

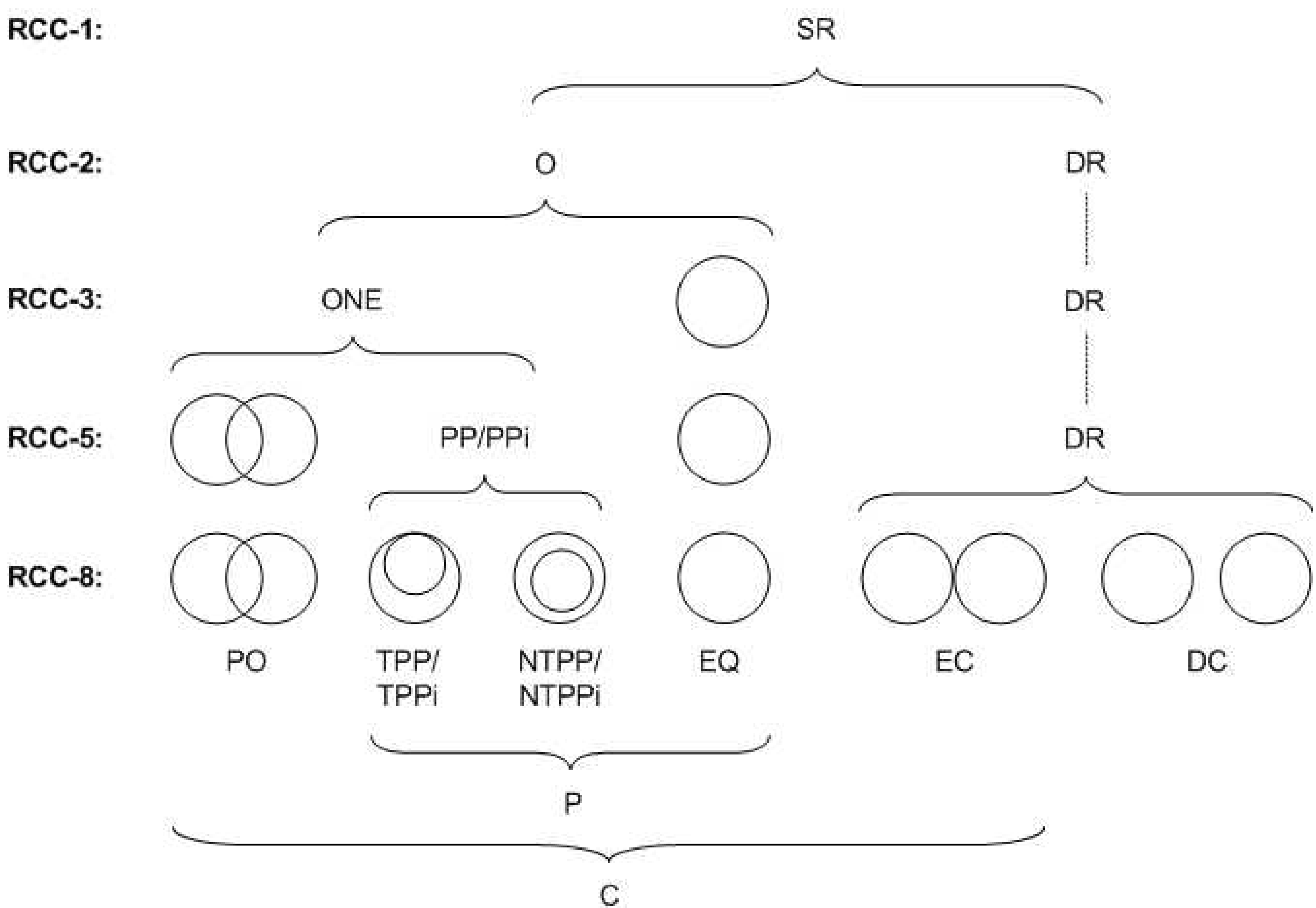
$TPP(x, y) \equiv_{def} PP(x, y) \wedge \exists z[EC(z, x) \wedge EC(z, y)]$



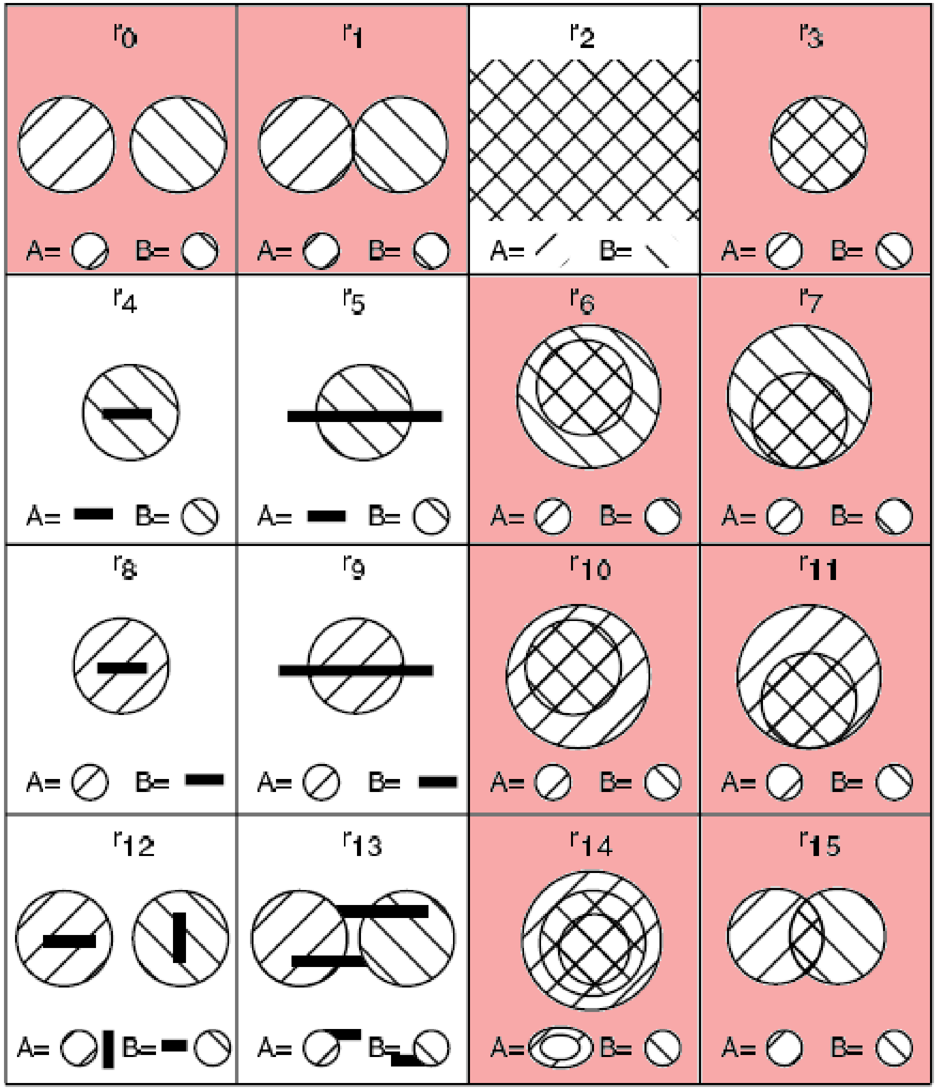
Inv. Tangential Proper Part of

$TPPi(x, y) \equiv_{def} TPP(y, x)$

RCC Family Tree



9-intersection model



- based on a **topological framework**
- hard to combine with DL
- subset can be interptreted as RCC-8

Egenhofer and Franzosa(1991)

RCC Relations as Concepts

NOT INTUITIVE

- Regions are non-empty regular closed sets.
- Translate RCC-8 relations into (sets of) concept axioms in OWL DL.
- Asserting for each concept an individual in the ABox
  - ensure classes cannot be empty
- Violate or workaround Type Separation

Katz and Grau (2005)
Stocker and Sirin (2009)

Axiomatize RCC relations in $\mathcal{SROIQ}$

- Axioms require **additional role constructors**:
  - intersection, complement.
- $\mathcal{SROIQ}$ supports complement but not intersection
- Language required with **extension of unrestricted form** of role inclusion axioms

$$S * T \sqsubseteq R_1 * \ldots * R_N$$

UNDECIDABLE

Grütter et al (2008)
Horrocks and Sattler (2003)
Patel-Schneider et al (2004)

RCC in Extended OWL-DL

- on level of **Knowlede Representation Systems**
- $\mathcal{SHOIN}$ augmented by RCC Rule Box
  - DL-safe rules for spatial relations
  - Complement by closing world
  - Intersection by composition tables

DECIDABLE HYBRID OWL-DL REASONER

Grütter at al (2008)
Motik et al (2004)

Consistency Checking

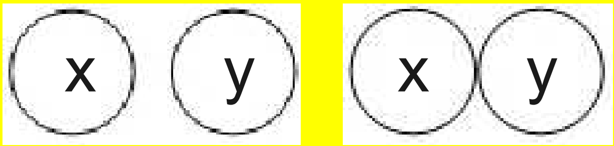
- Composition Table

- Uniform inference pattern

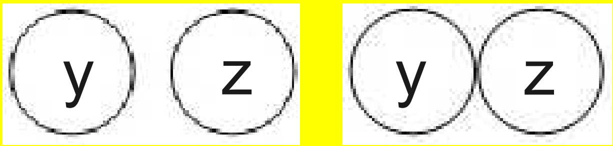
  $\forall x, y, z.\ S(x, y) \wedge T(y, z) \rightarrow R_1(x, z) \vee \ldots \vee R_N(x, z)$

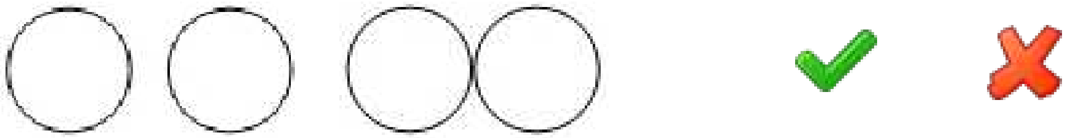
  $T(x, z) \equiv_{def} \{ DR(x, z), PO(x, z), EQ(x, z), PP(x, z), PPi(x, z) \}$

Compositions Examples

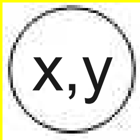


$DR(x,y) \wedge DR(y,z)$

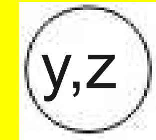


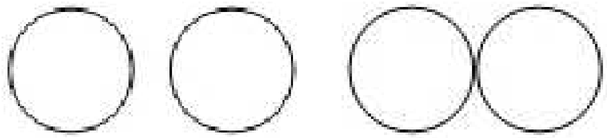
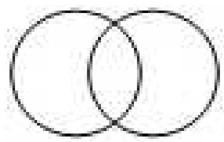
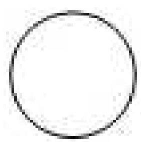
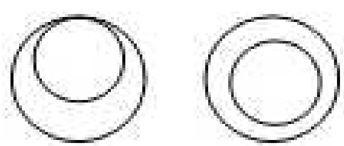
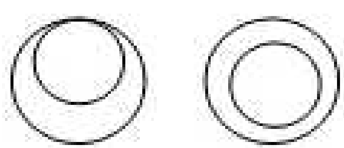
- DR(x,z) 
- PO(x, z) 
- EQ(x, z) 
- PP(x, z) 
- PPi(x, z) 

Compositions Examples

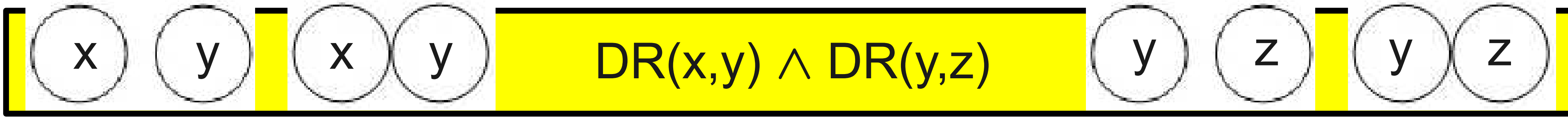


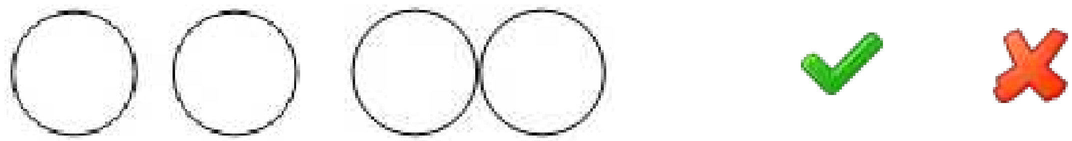
EQ(x,y) ∧ EQ(y,z)



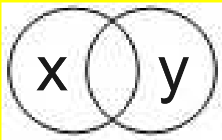
- DR(x,z) 
- PO(x, z) 
- EQ(x, z) 
- PP(x, z) 
- PPi(x, z) 

Compositions Examples

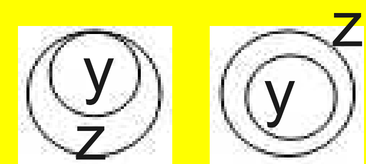


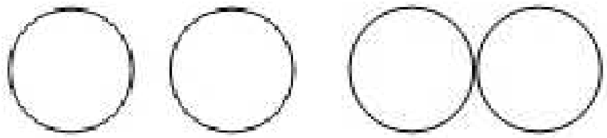
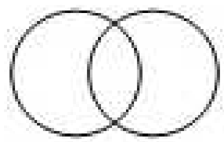
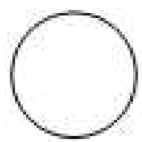
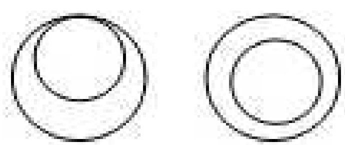
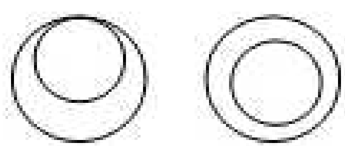
- DR(x,z) 
- PO(x, z) 
- EQ(x, z) 
- PP(x, z) 
- PPi(x, z) 

Compositions Examples



PO(x,y) ∧ PP(y,z)



- DR(x,z) 
- PO(x, z) 
- EQ(x, z) 
- PP(x, z) 
- PPi(x, z) 

Composition Table for RCC-5

EXAMPLE

*	DR(x,y)	PO(x,y)	EQ(x,y)	PPi(x,y)	PP(x,y)
DR(y,z)	T(x,z)	DR(x,z) PO(x,z) PPi(x,z)	DR(x,z)	DR(x,z) PO(x,z) PPi(x,z)	DR(x,z)
PO(y,z)	DR(x,z) PO(x,z) PP(x,z)	T(x,z)	PO(x,z)	PO(x,z) PPi(x,z)	DR(x,z) PO(x,z) PP(x,z)
EQ(y,z)	DR(x,z)	PO(x,z)	EQ(x,z)	PPi(x,z)	PP(x,z)
PP(y,z)	DR(x,z) PO(x,z) PP(x,z)	PO(x,z) PP(x,z)	PP(x,z)	PO(x,z) EQ(x,z) PP(x,z) PPi(x,z)	PP(x,z)
PPi(y,z)	DR(x,z)	DR(x,z) PO(x,z) PPi(x,z)	PPi(x,z)	PPi(x,z)	T(x,z)

$T(x, z) \equiv_{def} \{DR(x, z), PO(x, z), EQ(x, z), PP(x, z), PPi(x, z)\}$

So far so good

Take a deep breath...

...

Ready for part 2

Closing OWL-DL for RCC

APPROXIMATION
COMPLETE BUT
NOT SOUND

- Replace Equality with Subclass Inclusion (gives up soundness)

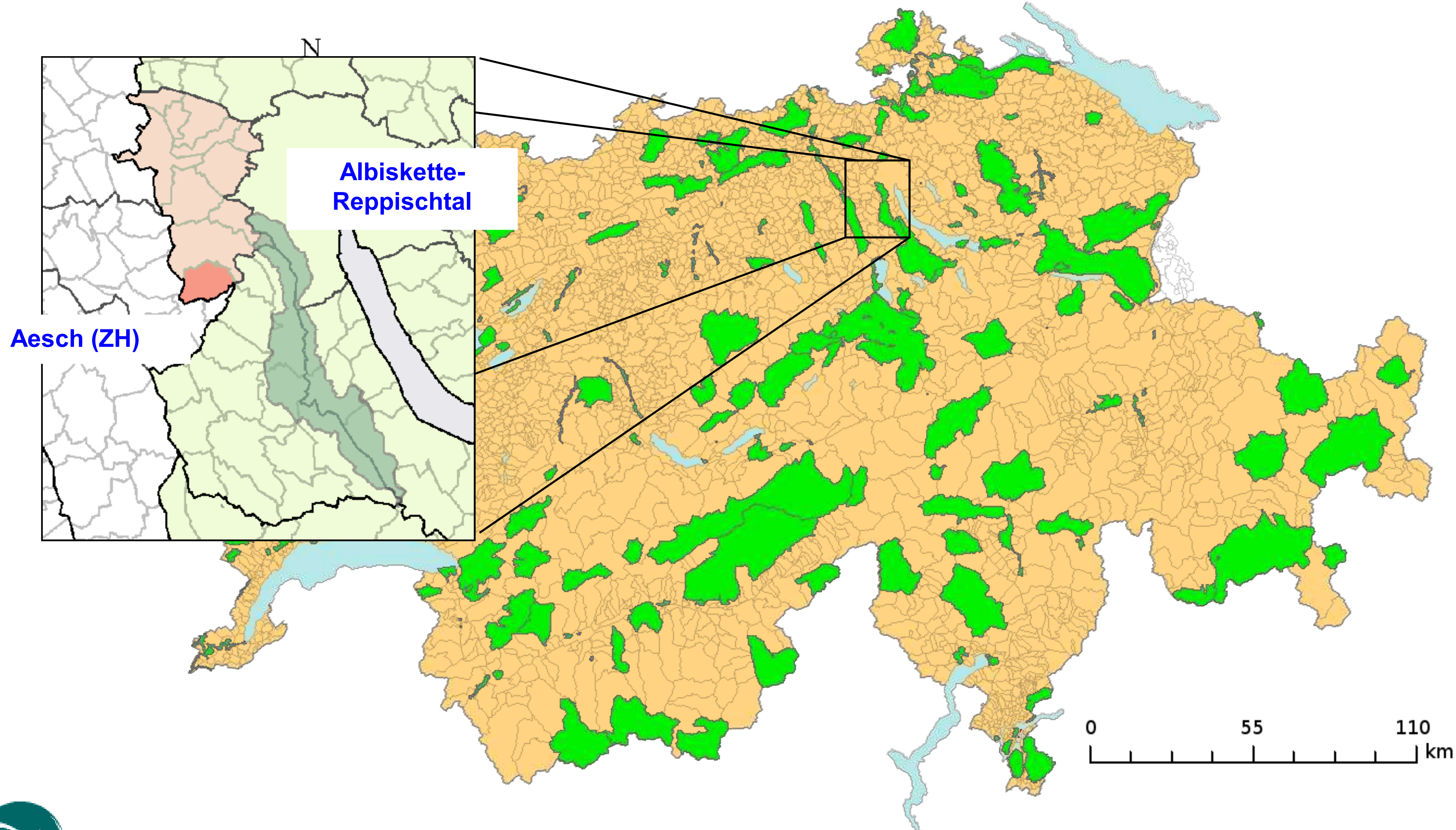
$$P(x, y) \equiv_{def} \forall z[C(z, x) \rightarrow C(z, y)].$$

$$P(x, y) \sqsubseteq \bigwedge z_i[C(z_i, x) \rightarrow C(z_i, y)] \; i=1, \ldots, N$$

Grütter et al (2008)
Grütter et al (2009)

Landscapes Close to Communities

EXAMPLE



Objective

given

- Geospatial world description
  - for assertion in the ABox of a knowledge base
  - based on attributes
    - which can easily be queried from spatial databases

inferred

- Construction and evaluation
  - of conjunctive queries
  - with thematic **and** spatial references
    - e.g., $\text{region} \sqcap \text{administrative} \sqcap \exists \text{overlaps.biotope}$

Region Connection Calculus

Basic theory

•assumes one primitive dyadic relation C(*x*, *y*)

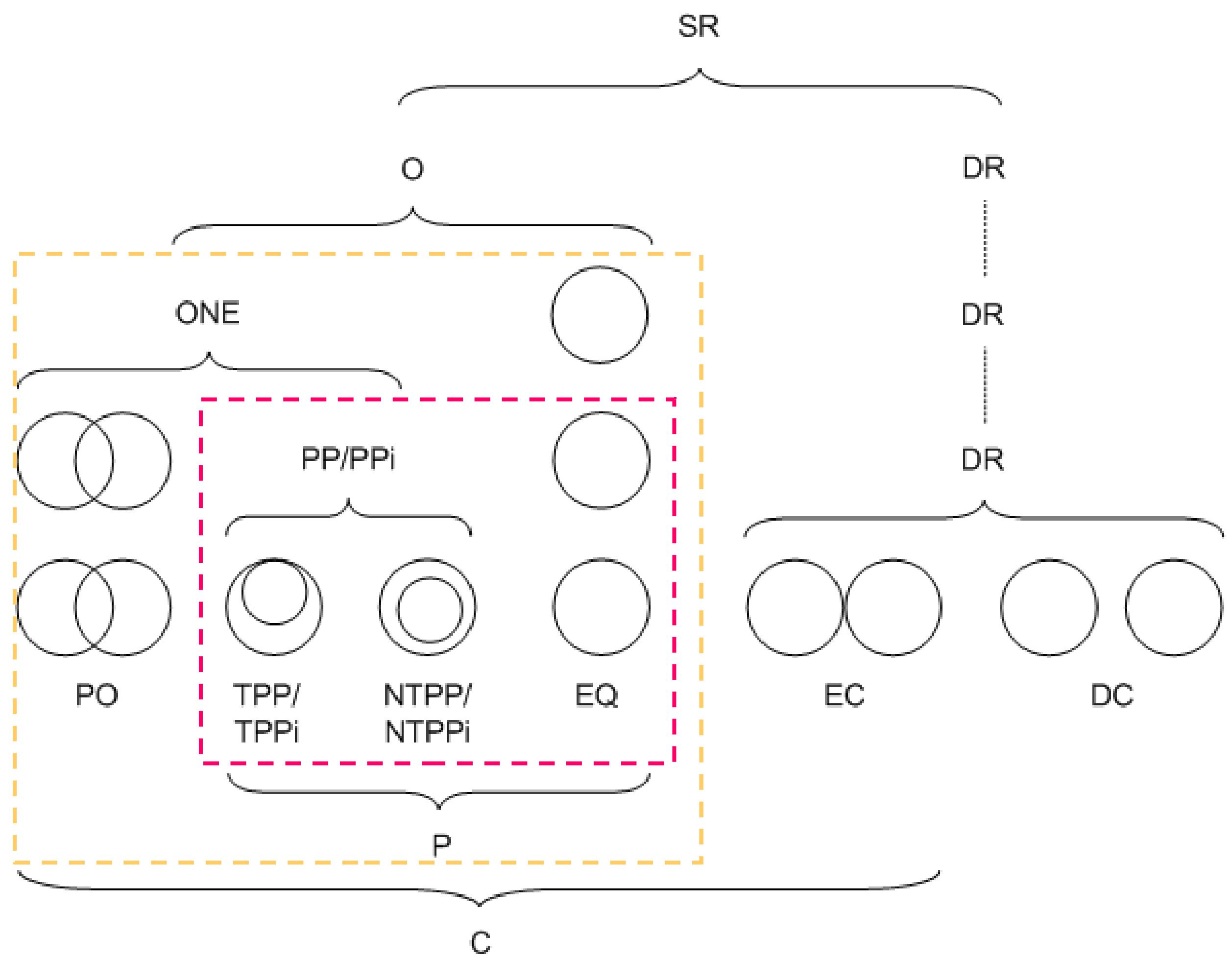
•individuals (*x*, *y*) can be interpreted as spatial regions

Using C(*x*, *y*)

•a number of significant relations can be defined e.g., P(*x*, *y*), O(*x*, *y*)

Spatial relations

•can be axiomatized in FOL



Closing the World for P(*x*, *y*)

$$P(x, y) \sqsubseteq \bigwedge z_i[C(z_i, x) \rightarrow C(z_i, y)]$$

with $1 \leq i \leq n$, n the number of regions represented

- universal quantifier replaced by **conjunction** ranging over all regions represented
  - *closed world* of a practical application
- equality sign is replaced by an **inclusion sign**
  - condition on the right hand side is *no longer sufficient* (but still necessary!)

A Calculus for P(*x*, *y*)

- is **not** expected to be sound
  - hypothesis for P(*x*, *y*) cannot be verified
- has been shown to be **complete**
  - hypothesis for P(*x*, *y*) can be falsified
- **approximates** a geospatial setting
  – how good is the approximation?
  – how can the approximation be controlled?

Geospatial Approximation

FUNCTION ***geospatialApproximation***

Input: hypothesis $T(., .)$, knowledge base KB = {ABox, TBox}

Output: *counter*

```
counter ← 0
M ← {(a, b)} ⊆ ABox
WHILE M is NOT empty
    SELECT (a, b) ∈ M
    IF T(a, b) is  NOT falsified in ABox THEN
        ABox ← ABox ∪ T(a, b)
        IF KB is consistent THEN
            counter ← counter + 1
        ELSE
            ABox ← ABox \ T(a, b)
        ENDIF
    ENDIF
    M ← M \ (a, b)
ENDWHILE
```

A Simple Open World TBox

0	$C \sqsubseteq \top$	$C^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
1	R	$R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
4	$\exists R.\top \sqsubseteq C$	$\{a \in \Delta^{\mathcal{I}} \mid \exists b.\ (a, b) \in R^{\mathcal{I}}\} \subseteq C^{\mathcal{I}}$
5	$\top \sqsubseteq \forall R.C$	$\Delta^{\mathcal{I}} \subseteq \{a \in \Delta^{\mathcal{I}} \mid \forall b.\ (a, b) \in R^{\mathcal{I}} \rightarrow b \in C^{\mathcal{I}}\}$
6	$S \sqsubseteq R$	$S^{\mathcal{I}} \subseteq R^{\mathcal{I}}$
7	$T \sqsubseteq S$	$T^{\mathcal{I}} \subseteq S^{\mathcal{I}}$

Proposition 1

- $T(a, b)$ in $\mathcal{A}_1 = \{C(a), C(b)\}$ -> $\mathcal{A}_1' = \{C(a), C(b), T(a, b)\}$ **consistent** w.r.t. $\mathcal{T}_1$ **= { 0, 1, 4, 5, 6, 7 }**.

Approximating a Closed World

2	$D \sqsubseteq C$	$D^{\mathcal{I}} \subseteq C^{\mathcal{I}}$
3	$D \sqsubseteq \neg(\exists S.D)$	$D^{\mathcal{I}} \subseteq \setminus \{a \in \Delta^{\mathcal{I}} \mid \exists b.\ (a, b) \in S^{\mathcal{I}} \land b \in D^{\mathcal{I}}\}$

Proposition 2

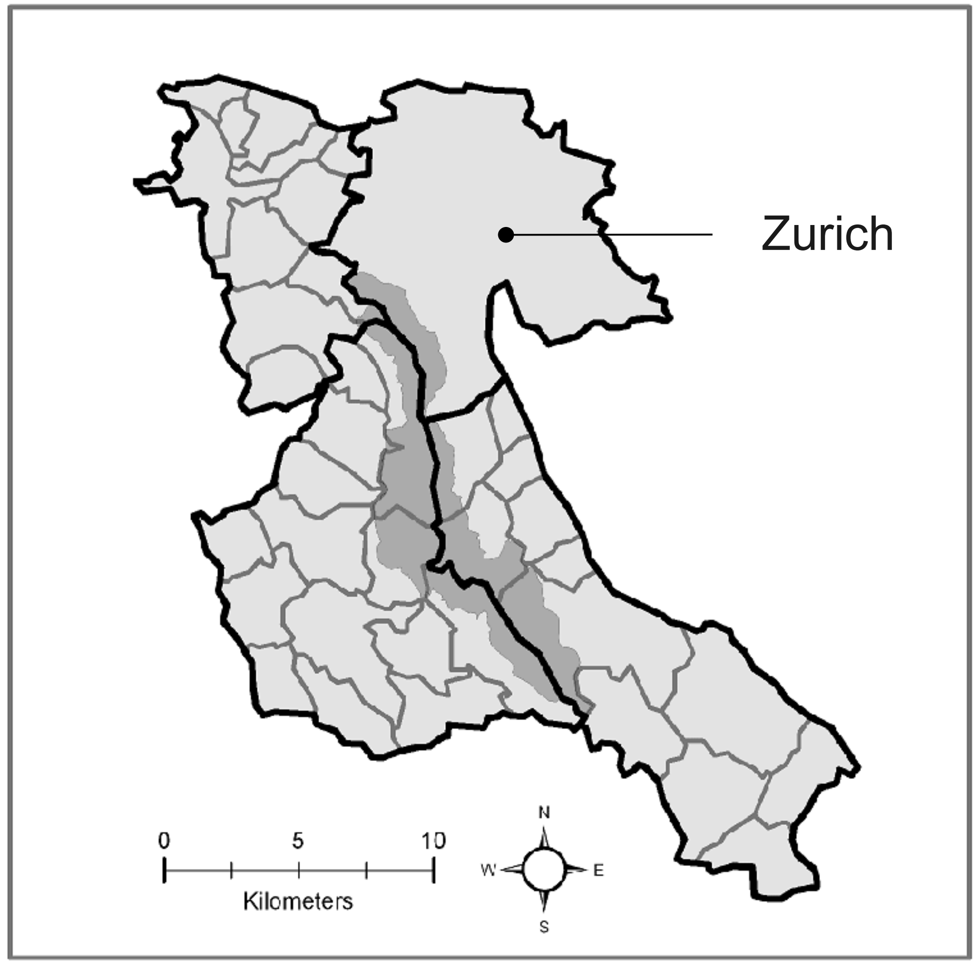
- $T(a, b)$ in $\mathcal{A}_2 = \{D(a), D(b)\}$ -> $\mathcal{A}_2' = \{D(a), D(b), T(a, b)\}$

  inconsistent w.r.t. $\mathcal{T}_1 = \mathcal{T}_2 \cup \{ 2, 3 \}$.

Applying the Approach

44 spatial regions
– from a productive GIS
– therefrom 38 communes
– asserted as individuals

262 relations
between connecting regions
– role assertions of type C(*x*, *y*)



Results

APPROXIMATION COMPLETE BUT NOT SOUND

- **Ground Truth:**
  - There exist 82 relations of type P(x,y)
- **Using TBox $\mathcal{T}_1$**
  - 109 relations of type P(*x*, *y*) are calculated
  - 27 relations are falsely calculated as P(*x*, *y*)
- **Using TBox $\mathcal{T}_2$**
  - 85 relations of type P(*x*, *y*) are calculated
  - only three relations are falsely calculated as P(*x*, *y*)
  - roughly ten times better than with $\mathcal{T}_1$

Discussion

EXTENSION TO OWL2

- Further improving the approximation
  - **qualified number restrictions**
    (communes may be part only of a single district)
  - further **refining model** in ABox
    - regions which are districts are asserted as such

OWL 2 feature

8	District $\sqsubseteq$ Region	$\text{District}^{\mathcal{I}} \subseteq \text{Region}^{\mathcal{I}}$
9	Commune $\sqsubseteq \leq 1$ partOf.District	$\text{Commune}^{\mathcal{I}} \subseteq \{a \in \Delta^{\mathcal{I}} \mid \lvert\{b \mid (a, b) \in \text{partOf}^{\mathcal{I}} \wedge b \in \text{District}^{\mathcal{I}}\}\rvert \leq 1\}$

Conclusion & Outlook

- Evaluating the approach on **different scales**
  - approximation is expected to be more precise with a high number of regions represented
  - impact of scalability on performance
- Extending the approach to **quality control**
  - improving imprecise computations
  - detection of erroneous data
- Applying the approach to **other domains**
  - identifying relevant problems in these domains

Take a deep breath...

...

OK, then let's start part 3

Defining Nearness in RCC

Qualitative description of nearness is based on a **qualitative representation** of distance!

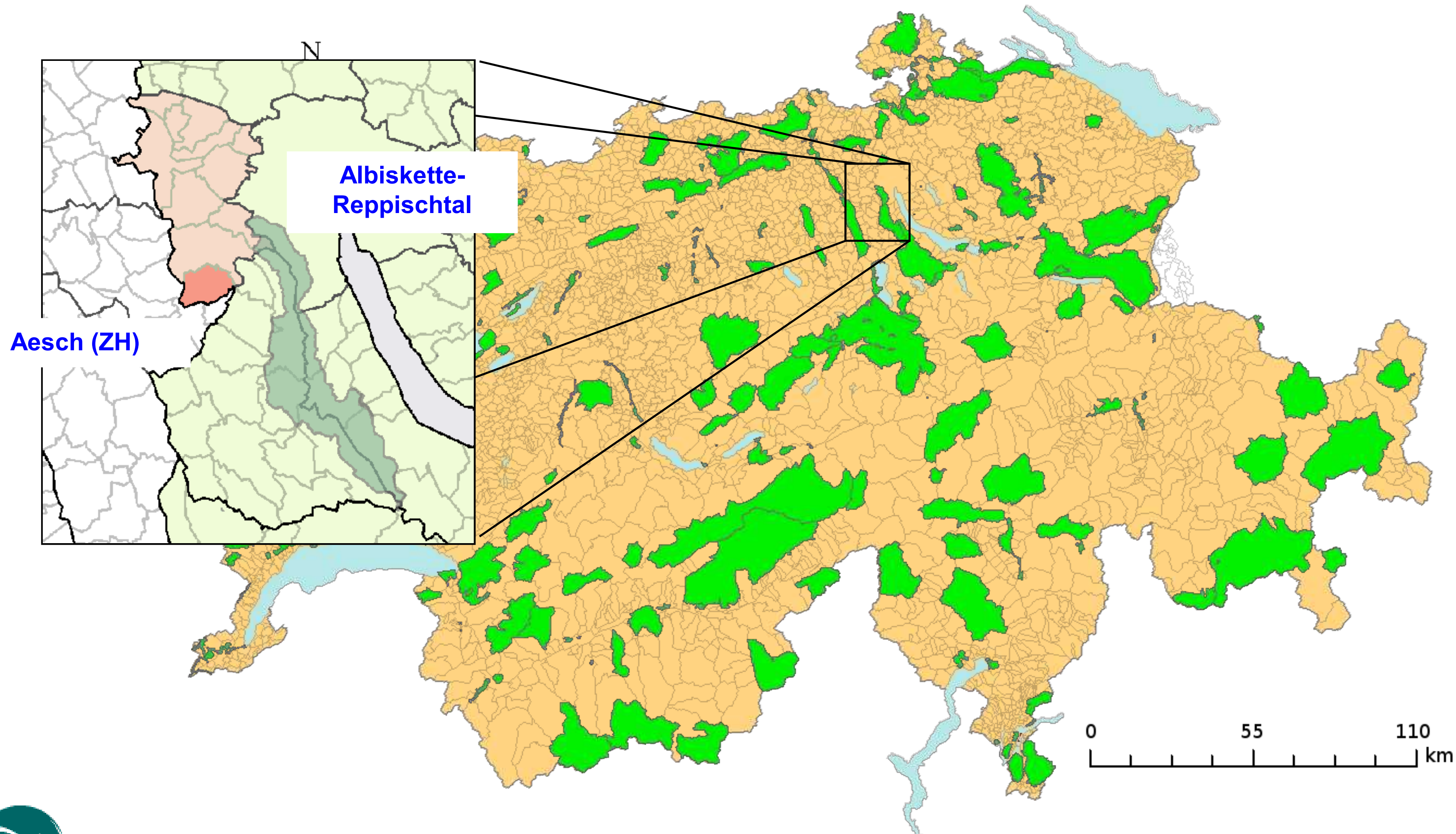
APPLICATIONS OF RCC

- Context dependent
- Scale sensitive
- Vague relation
- Weakly asymmetric relation

Brennan and Martin (2002)
Worboys (2001)
Hart and Dolbear (2006)
Grütter el al. (2010)

Finding Landscapes
Close to Communities

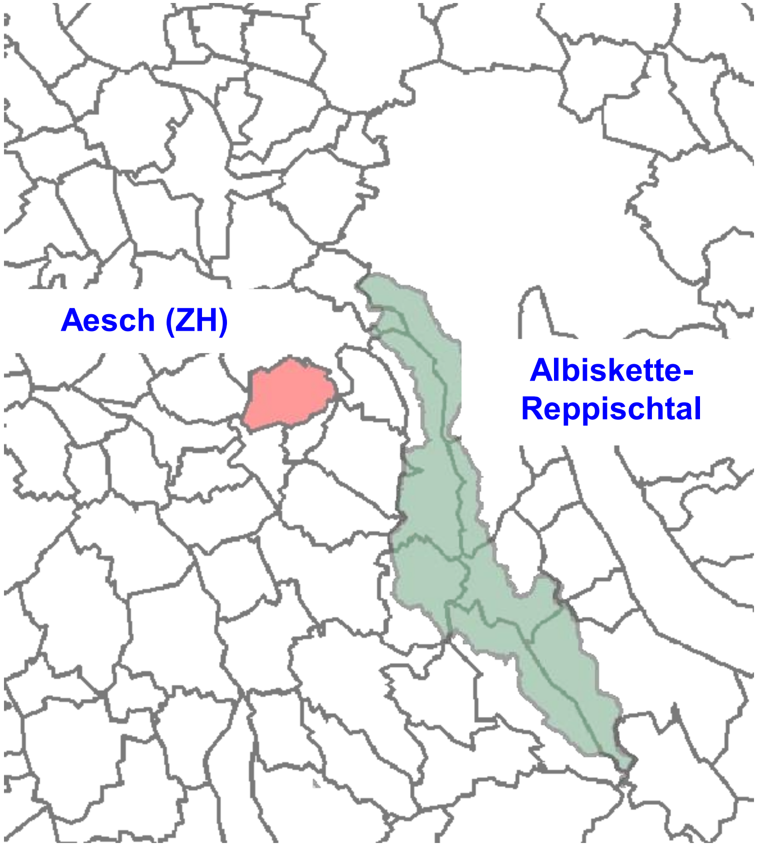
EXAMPLE



Landscapes Close to Aesch (ZH)

EXAMPLE

- `Albiskette-Reppischtal` is a landscape of national importance potentially close to `Aesch (ZH)`
- `<Landschaften "in der Nähe von" "Aesch (ZH)">` Returns 1’350 matches
  - None of top 30 deal with a landscape of national importance
- `<Albiskette-Reppischtal>` Returns 230 matches
  - None appear among top 30 of initial search



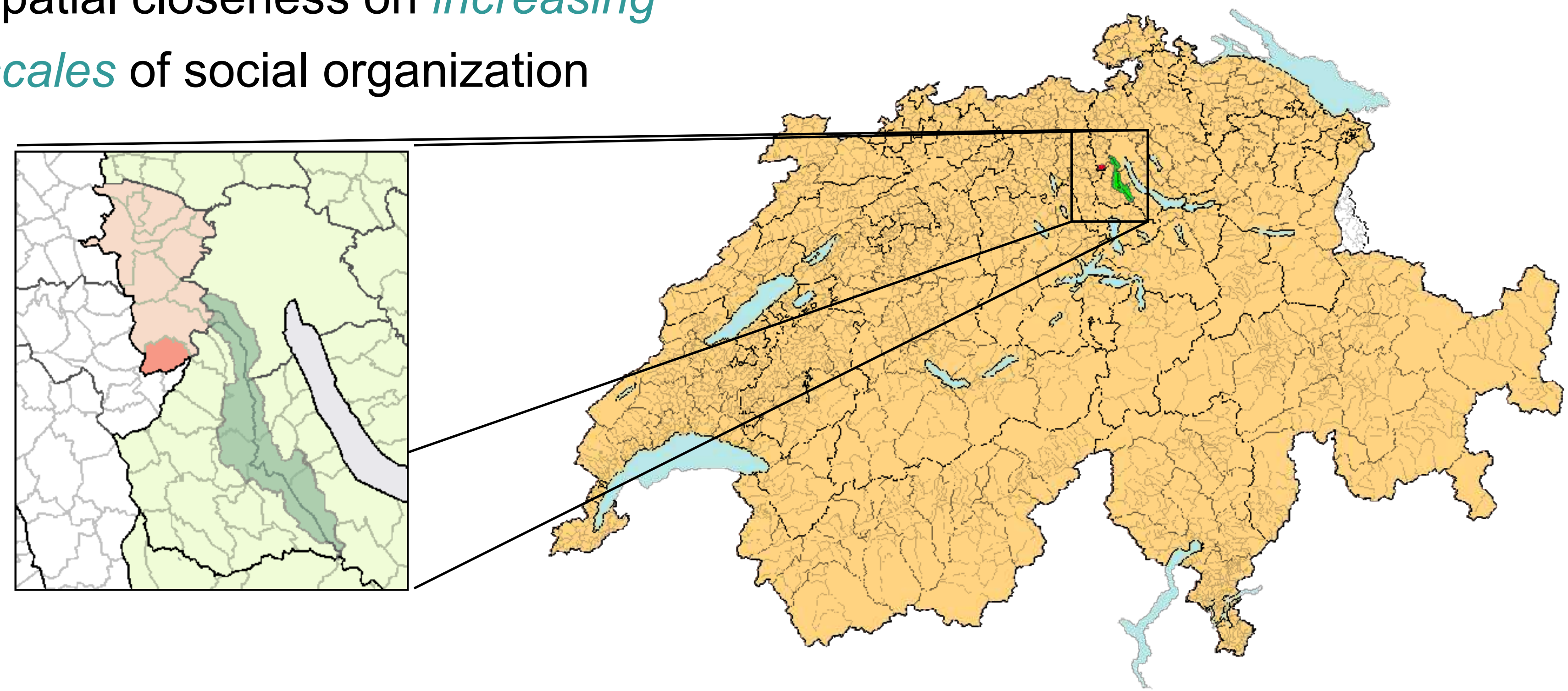
Representing Spatial Knowledge

- Representing spatial knowledge
  - Region Connection Calculus (RCC)
- *Extending RCC* such as to include “close to”
- Implementing RCC in *OWL DL and DL-safe rules*
  - Knowledge Base ($\mathcal{KB}$)
  - Rule Base ($\mathcal{RB}$)
- Processing (possibly vague) *spatio-thematic* queries

Partitions in RCC

EXAMPLE

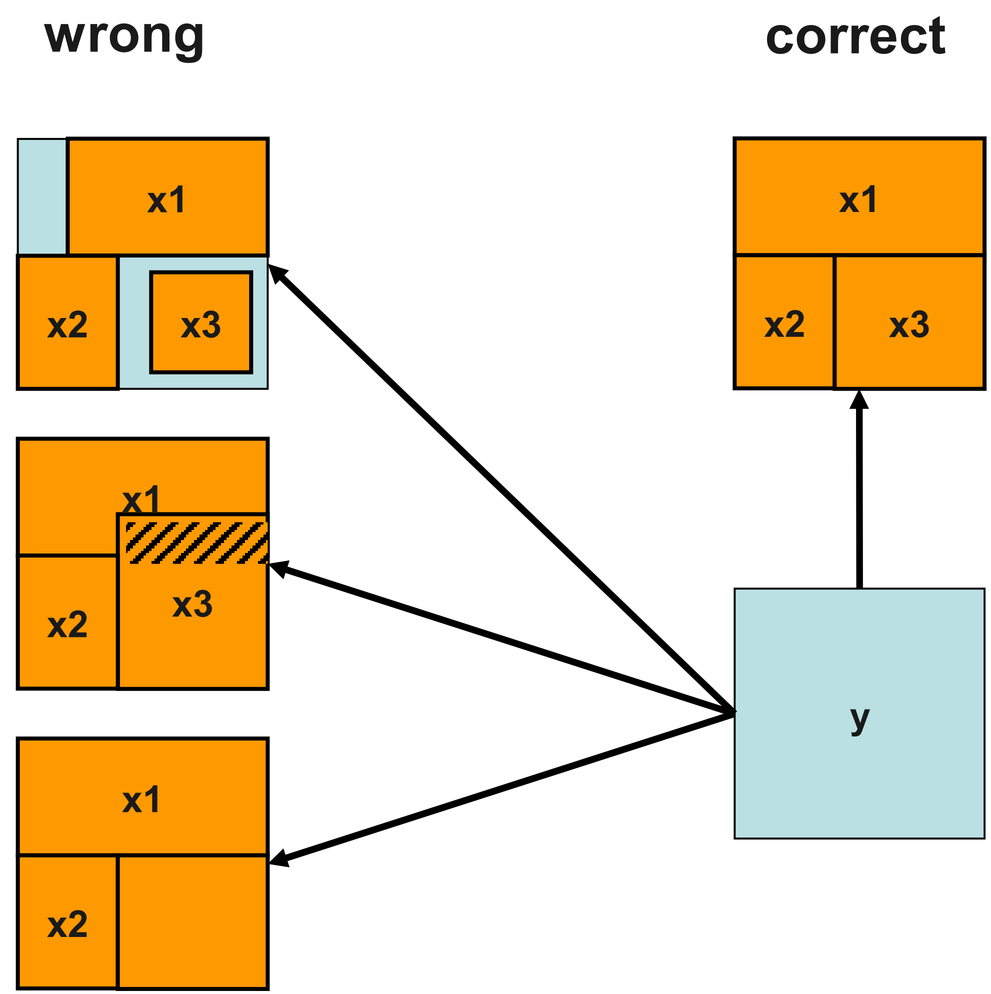
- Administrative regions are *social artifacts*
  - Mirror how a collective perceives spatial closeness on *increasing scales* of social organization
- Administrative regions are organized in *partitions*



Family of regions $(x_i)_{i \in I}$ partition of y

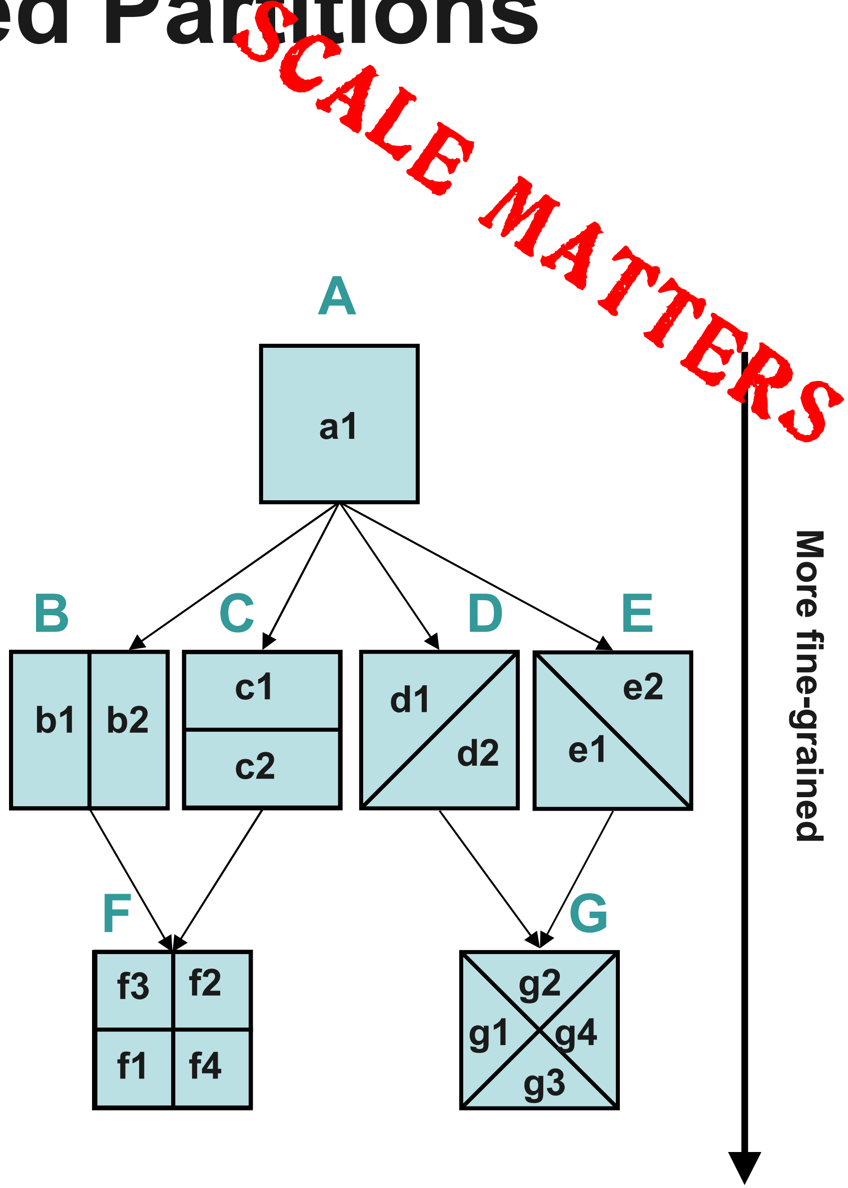
REPRESENTING SPATIAL KNOWLEDGE

- $y = \mathrm{SUM}_{i \in I}\, x_i$
  for **finite index set *I***
- $\forall x_i\ \forall x_j\ R(x_i, x_j)$ for $i \neq j$
- **regions** $(x_i)_{i \in I}$ **are named** for all index sets ***I***



Partial Order on Typed Partitions

- Distinguish partitions by *typed* elements
  - Example: Community(x_i) says that x_i is of type Community
- Different scales by *partial order* on partitions
  - reflexive, transitive and antisymmetric.
  - Example: Community$(x_i)_{i \in I} \preceq$ District$(y_j)_{j \in J}$



Minimal Partial Order on Typed Partitions

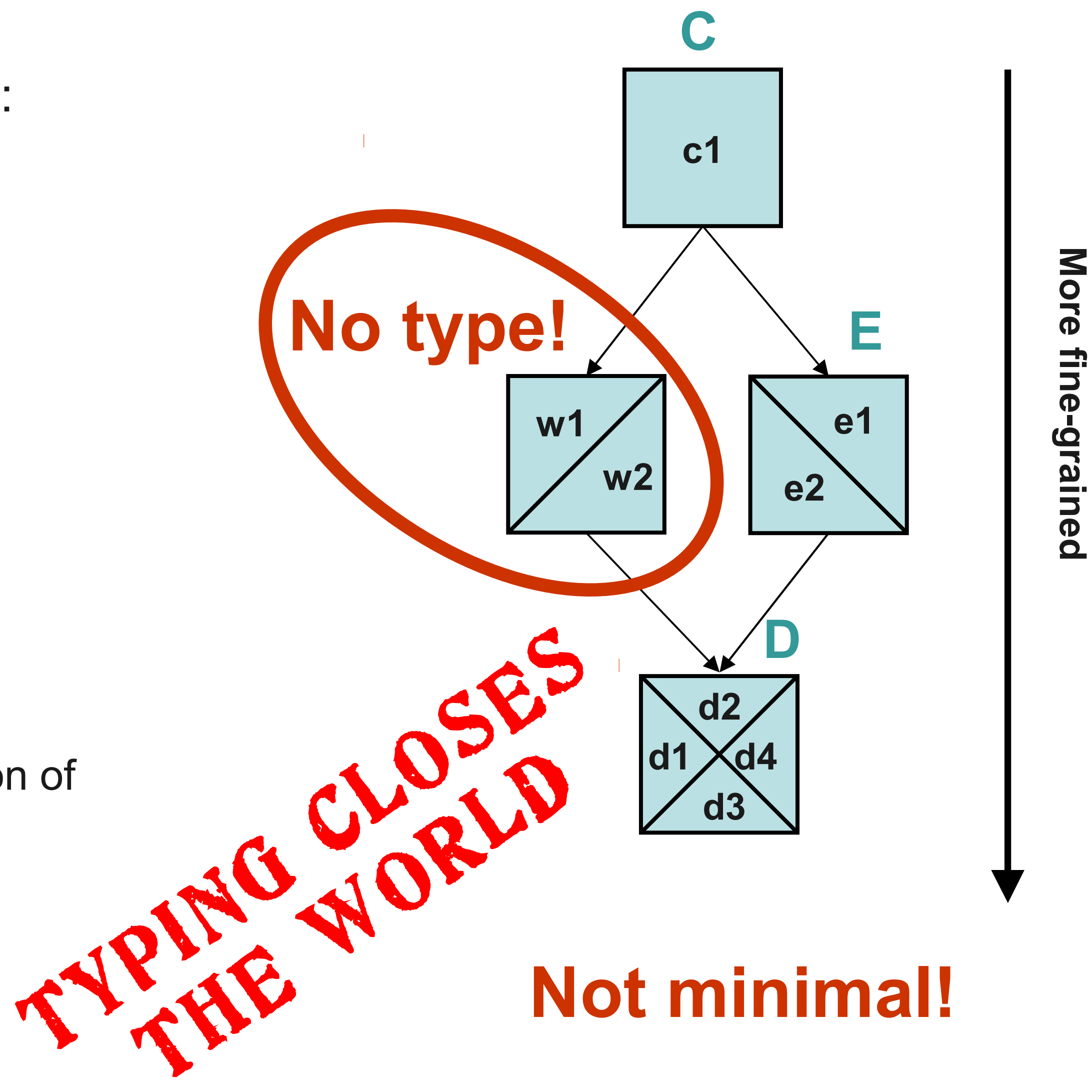
- A *minimal* partial order (m.p.o.) in RCC links to partial order in conceptualization:

  If $C(x_i)_{i \in I} \preceq (w_k)_{k \in K} \preceq D(y_j)_{j \in J}$,

  then $(w_k)_{k \in K}$ must by typed

- m.p.o. on typed partitions is *intransitive*.

- Example:
  - Conceptualization provides administrative types District and Commune.
  - Partial order comprising non-typed partition of intermediate granularity is not minimal.



Defining Closeness in RCC

EXAMPLE

- Closeness in RCC can be **inferred by composition rule**

  $\forall x_i \forall y_j \forall z\ [P(x_i, y_j) \wedge XC(z, y_j) \rightarrow CL(z, x_i)]$

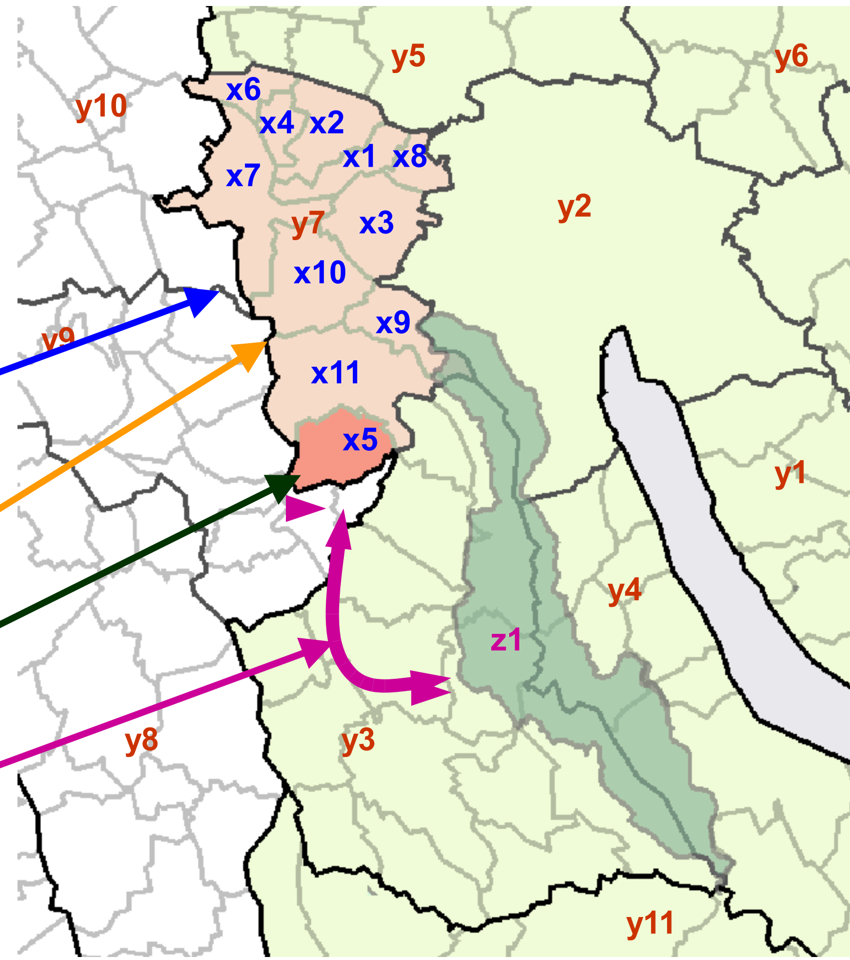
  – $XC(z, y_j) \leftrightarrow C(z, y_j) \wedge \neg Pi(z, y_j)$

- Example:

  – $Community(x_i)_{i \in I} \preceq_{min} District(y_j)_{j \in J}$

  – $Community(x_5)$

  – $P(x_5, y_7) \wedge XC(z_1, y_7) \models CL(z_1, x_5)$



DL Knowledge Base and RCC-Rule Base

- Knowledge Base $\mathcal{KB}$
  - P(x_i, y_j) and subrelations are *functional roles*:
    an individual x_i is only part of a single region y_j
  - Partitions are *nominals*
  - *m.p.o.* on typed partitions:
    asserting partOf(x_i, y_j) and subrelations,
    exclusively for (x_i, y_j) with $C(x_i)_{i \in I} \preceq_{min} D(y_j)_{j \in J}$
- RCC-Rule Base $\mathcal{RB}$
  - Composition as *DL-safe rule*

IMPLEMENTATION IN OWL-DL

Closing OWL-DL for RCC

SOUND & COMPLETE

- Type Sets of Regions
  - Nominals

    $C \sqsubseteq \{x_1, \ldots, x_n\}$

- Prohibit transitivity of spatial roles
  - partial order on typed sets

Grütter et al (2008a)
Grütter et al (2009)

Summary

- Couple **geometrical computations** with

  symbolic reasoning services

- RCC allows **inferring implicit knowledge**

  from the knowledge **explicitly represented**.

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