UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO

DIPARTIMENTO DI INFORMATICA

PhD Program in Computer Science and Mathematics
XXXIII cycle

Research Project

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1. Research Title

Long time decay estimates for dissipative hyperbolic and pseudo-hyperbolic equations

2. Research Area

Partial Differential Equations

3. Research motivation and objectives

The research will address several topics related to the study of dissipative equations:

- Decay estimates will be obtained for damped Klein-Gordon equations with time-dependent coefficients in the form

$$\begin{cases} u_{tt} - a(t)\Delta u + b(t)u_t + m(t)u = 0 \quad t \geq 0,\ x \in \mathbb{R}^n \\ u(0,x) = u_0(x), \\ u_t(0,x) = u_1(x). \end{cases}$$

  A special emphasis will be given to how the interplay between the coefficients lead to different asymptotical properties of the solution. In particular, the diffusion phenomenon to the corresponding heat equation

$$\begin{cases} b(t)v_t - a(t)\Delta v + m(t)v = 0 \quad t \geq 0,\ x \in \mathbb{R}^n \\ v(0,x) = v_0(x), \end{cases}$$

  will be investigated.
- $L^r - L^q$ estimates, $1 \leq r \leq q \leq \infty$ will be derived for dissipative higher order pseudo-hyperbolic equations in the form

$$\begin{cases} A_2 u_{tt} + A_1 u_t + A_2 u = 0 \quad t \geq 0,\ x \in \mathbb{R}^n \\ u(0,x) = u_0(x), \\ u_t(0,x) = u_1(x). \end{cases}$$

  In particular, the cases where $A_2 = \mathrm{I}, \mathrm{I} - \Delta,\ A_1 = \mathrm{I}, -\Delta, \Delta^2,\ A_0 = -\Delta, \Delta^2, \Delta^2 - \Delta$ will be investigated. These cases includes several equations from theory of elastic waves, the beam equation, the plate equation, the double diffusion equation, waves with structural or viscoelastic damping. Phenomena like the regularity-loss type decay, the effectiveness of the dissipation will be studied.
- Equations with fractional in time derivatives and integrals will be object of the last part of the research. Recently, there has been a growing interest for equations which contain these terms, since the presence of integration in time allows to *keep alive the story of the system,* and so better describe the influence from forces and reactions in real-world applications. The first model of this type is given by the fractional diffusive equation

$$D^{\alpha}_{0t} v + Av = 0, \qquad t \geq 0,\ x \in \mathbb{R}^n.$$

- The information on the profile of the solution to the previous problems will allow to determinate the critical exponent for semilinear problems with power nonlinearities $f(u, \nabla u, u_t)$ and possibly with nonlinear memory terms. By critical exponent we mean the threshold power p such that global existence of small data solutions holds in the supercritical case, and no global solutions existence, under suitable sign assumption on the data, in the subcritical one.

4. State-of-the-art

The use of information about the asymptotic profile of the solution to a dissipative wave equation to study semilinear problems goes back to [61], who investigated the damped wave equation

$$\begin{cases} u_{tt} - \Delta u + u_t = 0 \quad t \geq 0,\ x \in \mathbb{R}^n \\ u(0,x) = u_0(x), \\ u_t(0,x) = u_1(x), \end{cases}$$

and applied the obtained decay estimates to study the problem with **smooth** power nonlinearity $f(u, \nabla u, u_t)$. In 2001, G. Todorova and B. Yordanov [86] proved global existence of small data solution for the semilinear damped wave equation,

$$\begin{cases} u_{tt} - \Delta u + u_t = |u|^p, \quad t \geq 0, x \in \mathbb{R}^n, \\ (u, u_t)(0,x) = (u_0, u_1)(x), \end{cases}$$

in the supercritical range $p > 1 + 2/n$, by assuming small data in weighted energy space. In particular, power nonlinearities close to the critical one are not $\mathcal{C}^2$ in space dimension $n \geq 3$. G. Todorova and B. Yordanov also proved nonexistence of global solutions in the subcritical case $1 < p < 1 + 2/n$, whereas it was proved in the critical case in [91]. By only assuming data in Sobolev spaces, the existence result was proved in space dimension $n \leq 5$ in [70], by using $L^r - L^q$ estimates, $1 \leq r \leq q \leq \infty$.

Indeed, the main difference with respect to the wave equation with no dissipation, is that the damping term u_t produces the *diffusion phenomenon.* This effect modifies the asymptotic profile of the solution to the corresponding linear problem so that it can be described by the solution to a heat equation with suitable initial data (see [37] and, later, [36, 58, 72]):

$$\begin{cases} v_t - \Delta v = 0, \quad t \geq 0, x \in \mathbb{R}^n, \\ v(0,x) = u_0(x) + u_1(x). \end{cases}$$

One important difference with respect to the wave equation with no dissipation is also that the critical exponent for this latter does not come from scaling arguments. The critical exponent for

$$\begin{cases} u_{tt} - \Delta u = |u|^p, \quad t \geq 0, x \in \mathbb{R}^n, \\ (u, u_t)(0,x) = (u_0, u_1)(x), \end{cases}$$

is $p_0(n)$, the positive solution to

$$(n-1)\,p^2 - (n+1)\,p - 2 = 0.$$

More precisely, if $1 < p \leq p_0(n)$, then solutions blow-up in finite time for a suitable choice of initial data (see [35], [47], [46], [78], [79], [87]), whereas for $p \in (p_0(n), (n+3)/(n-1)]$ a unique global (in time) small data solution exists (see [33], [32], [47], [84], [92]). In space dimension $n = 1$, solutions blow-up in finite time for any $p > 1$, hence, we put $p_0(1) = \infty$ (see [35]).

In [23], the model with time-dependent damping

$$(1)\qquad \begin{cases} u_{tt} - \Delta u + b(t)u_t = |u|^p, & t \geq 0,\ x \in \mathbb{R}^n, \\ u(0,x) = f(x), \quad u_t(0,x) = g(x), \end{cases}$$

has been considered, and it has been proved that the critical exponent for global small data solutions to (1) remains the same of the problem with $b = 1$ under the assumption of *effectiveness* of the damping term [90] on the estimates for the corresponding linear problem:

$$(2)\qquad \begin{cases} u_{tt} - \Delta u + b(t)u_t = 0, & t \geq 0,\ x \in \mathbb{R}^n, \\ u(0,x) = f(x), \quad u_t(0,x) = g(x). \end{cases}$$

In particular, global existence holds for $p > 1 + 2/n$ if initial data are assumed to be small in exponentially weighted energy spaces. In the subcritical and critical range, $1 < p \leq 1 + 2/n$, no global small data solutions exist, under a suitable sign assumption [20]. In the case of polynomial shape $b(t) = \mu(1+t)^k$, the damping is effective if $k \in (-1, 1]$ (see [56, 73, 89] for the corresponding global existence result), and partially effective if $k = -1$.

In this latter case $b(t) = \mu(1+t)^{-1}$, the critical exponent of global small data solutions to (1) remains $1 + 2/n$ if μ is sufficiently large [11, 88], whereas it seems to increase to $\max\{p_S(n+\mu), 1 + 2/n\}$, as μ becomes smaller with respect to the space dimension, as conjectured in [21, 24] (see also [39, 53]), where p_S is Strauss exponent for the semilinear undamped wave equation [33, 47, 82]. The overdamping case $b(t) = \mu(1+t)^k$, with $k > 1$ has been studied in [40].

The study of higher order evolution equations has also a special interest for physical phenomena. Models to study the vibrations of thin plates ($n = 2$) given by the full von Kármán system have been studied by several authors, in particular see [8, 51, 54, 55, 77]. Perla Menzala and Zuazua [62] considered the full von Kármán system and they proved that the Timoshenko's model

$$u_{tt} - \gamma \Delta u_{tt} + \Delta^2 u + u = 0, \qquad t \geq 0,\ x \in \mathbb{R}^2,$$

may be obtained as limit of a full von Kármán system when suitable parameters tend to zero. The term $-\Delta u_{tt}$ in the previous equation is to absorb in the system the rotational inertia effects. It is well known that the plate equation with such term is a hyperbolic equation with finite speed of propagation, whereas the non-rotational plate model has infinite speed of propagation. In particular, a structural damping of type $-\triangle u_t$ has been considered in [2]. The asymptotic profiles for the plate equation with a generalized rotational inertia term have been derived in [38].

The plate equation for small data with rotational inertia, damping term u_t and nonlinearity $|u_t|^p$, has been investigated in [3, 83]. Linear estimates, which are a fundamental tool to attack semilinear problems, have been derived for damping term of type $b(t)\,(-\Delta)^\theta u_t$ in [12]. We mention that the nonlinear plate equation, in absence of the damping term, has been studied in [22].

Finally, semilinear fractional diffusive equation [30]

$$D^\alpha_{0t} u - \Delta u = |u|^p,$$

for which Caputo fractional derivative in time appear, has been very recently studied in [17] by using the Mittag-Leffler functions to write the Fourier transform of the fundamental solution of the problem. We refer to [50] for an introduction on the theory of fractional derivatives and to [1, 28, 52, 57, 59, 60] to illustrate some applications.

5. Problem approach

The main tool in the study of the problems of the project rely on the application of the Fourier transform to the equation. In some cases, the solution of the corresponding ODE problem will be explicitly determined, in other cases it will be possible to obtain only suitable qualitative properties. Either mapping properties of the Fourier transform or multiplier theorems will then be used to derive estimates of the action of the solution operator in Sobolev spaces with respect to the space variable, emphasizing the dependence on time as $t \to \infty$. Gagliardo-Nirenmberg inequalities and other functional inequalities will be used to study the nonlinear problem, obtaining global small data solutions via contraction arguments, and nonexistence of global solutions via the test function method or suitable functional methods.

6. Expected results

The research aims to the study of several examples of dissipative equations; being able to explain the influence of several terms of the equation on its asymptotic profile or critical nonlinear power allows to understand the influence that the interplay of different forces and reactions have on the unknown of the problem, in the physical world. In particular, dissipative behaviors and control of the influence of nonlinear terms, are strongly connected to the stability of physically meaningful solutions.

Also the study of higher order evolution equations has a special interest for physical models. For instance, fourth-order evolution partial differential equations arise in problems of solid mechanics as, for example, in the theory of thin plates and beams. In particular formulations of problems, related with the Navier-Stokes equations (see [85]), are even described by elliptic equations of fourth-order. Possibly, systems coming from linearized equations of magneto hydro dynamics will be studied, as well.

7. Phases of the project

The realization of the project is not differentiated in phases, as it is customary in pure mathematics projects on PDEs from young researchers.

There is no need of special resource to take on the project, besides the access to an academic library and the possibility to occasionally participate to conferences, workshops, and to do scientific visits to collaborators in other universities.

It is expected that at the end of the each year of the Ph.D. course, approximately a preprint paper on one of the project topics will be produced. Possibly, one or two of them will be accepted for publication before the end of the third year. Possibly, during the second or the third year, the researcher will be invited to disseminate his results to an international conference.

8. Result evaluation

The best way to evaluate the obtained results consists in reading the produced papers and preprints. However, the evaluation of the impact factor of the journals were papers will be accepted for publication, gives an indirect method to get an idea of the quality of the results, even with all limits of the employment of this system to evaluate very young researchers. Another way to evaluate the success of the research consist in check if the researcher will be invited to give a talk about his results in local or international conferences or workshops.

9. Possible reference persons external to the department

- Prof. Marcelo R. Ebert (University of São Paulo) is an expert in asymptotic profile for linear pseudo-hyperbolic equations.
- Prof. Vladimir Georgiev (University of Pisa) is an expert about nonlinear hyperbolic PDEs.
- Prof. Tohru Ozawa (Waseda University) is an expert about nonlinear hyperbolic PDEs.
- Prof. Michael Reissig (University of Freiberg) is an expert in evolution equations with time-dependent coefficients.

10. References

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