

PARTIAL DIFFERENTIAL EQUATIONS

Ph.D. in Informatics and Mathematics

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Introduction to Conservation Laws.

Scalar conservation laws. System of conservation laws. Examples and applications.

The Method of Characteristics.

Semilinear equations with constant coefficients. Semilinear equations with variable coefficients.

Quasilinear equations.

Entropy Weak Solutions.

Discontinuous solutions. Rankine-Hugoniot conditions. Entropies and entropy fluxes. Entropy weak solutions. Liu conditions. Kruzhkov Theorem. Uniqueness and stability of entropy weak solutions. Change of coordinates.

Riemann Problem.

Solution of the Riemann problem for convex fluxes. Solution of the Riemann problem for general fluxes.

Functions with Bounded Variation.

Definition. Examples. Approximation via piecewise constant functions. Compactness.

Front-tracking.

Functions with bounded variation. Construction of front-tracking approximants. Existence of entropy weak solutions for the Cauchy problem in BV .

Vanishing Viscosity.

Vanishing viscosity approximants. Bardos Lemma. BV Estimates. Existence of entropy weak solutions for the Cauchy problem in BV .

Compensated Compactness.

Young measures. Murat Lemma. Div-Curl Lemma. Tartar Theorem. Application to conservation laws. Existence of entropy weak solutions for the Cauchy problem in $L^1 \cap L^\infty$.

Periodic Solutions.

Well-posedness of periodic entropy solutions. Asymptotic decay.

Oleinik Estimate.

Oleinik estimate for conservation laws with convex fluxes. Uniqueness via Oleinik type estimates.

Lax-Oleinik Formula.

Legendre transform. Lax-Oleinik Formula. Existence of entropy weak solutions for the Cauchy problem in L^∞ .

SUGGESTED TEXTBOOKS

- [1] A. Bressan. Hyperbolic Systems of Conservation Laws. The one-dimensional Cauchy problem. Oxford Lecture Series in Mathematics and its Applications, vol. 20, Oxford University Press, Oxford, 2000.
- [2] C. M. Dafermos. Hyperbolic Conservation Laws in Continuum Physics. Second edition. Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], vol. 325, Springer-Verlag, Berlin, 2005.
- [3] L. C. Evans. Partial differential equations. Graduate Studies in Mathematics, vol. 19, American Mathematical Society, Providence, RI, 1998.

- [4] H. Holden and N. H. Risebro. Front tracking for hyperbolic conservation laws. Applied Mathematical Sciences, vol. 152, Springer-Verlag, New York, 2002.
- [5] D. Serre. Systems of conservation laws - 1 & 2. Cambridge University Press, Cambridge, 2000.