Convex Analysis and Optimization

Teacher
Andersen Ang (https://angms.science/index.html)

Course description (min 150, max 300 words)
This module is about the fundamentals of algorithms solving continuous optimization problems, which involve minimizing functions of multiple real-valued variables, possibly subject to restrictions, constraints, and nondifferentiable regularisations on the values that the variables may take. We focus (not exclusively) on convex optimization, where the choice of topics is motivated by relevance to machine learning and data science.

The module has a two-part syllabus.
Part 1 covers the theoretical foundation of optimization: convex analysis. Topics include the notion of convexity, subdifferential, optimality conditions, and properties of various formulations of continuous optimization problems.

Part 2 focuses on methods for solving optimization problems. Topics include various gradient descent methods, higher-order methods, coordinate descent, randomization, and heuristics.

[Module focus] This module is on structural continuous nonlinear optimization in the finite-dimensional real Euclidean space. This module is not about linear programming, combinatorial optimization nor PDE-constrained optimization.

[Prerequisites] A good knowledge of linear algebra and (differential) calculus is required for this module. Exposure to numerical analysis and vector calculus is helpful but not required; the applications will be kept basic and simple. Familiarity with programming is required.

[Who should enroll] This module is expected to be beneficial to anyone who uses or will use optimization in machine learning and related work. More specifically, people from the following fields: machine learning, signal and image processing, communications, bioinformatics, control, robotics, computer graphics, computer vision, operation research, scientific computing, computational mathematics, and finance.

[Content]
- Foundational Topics
- Differential Calculus in Finite-dimensional Euclidean Space
- Convex Set
- Convex Function
- Subdifferential and Optimality Condition
- Gradient Descent
- Proximal Operators and Projection
- Lagrangian Methods, Duality
- ADMM
- Acceleration in First-order methods

- Second-order Methods
- Coordinate Descent
- Stochastic Gradient Methods
- (Optional) Vector Optimization
- (Optional) Optimization as Differential Equations
- (Optional) Optimization as Monotone Operator

Course period
January 2025

SSD
MAT/08

Course References (optional)

Credits and Hours
2,5 credits (20 hours)

Exam Modality
Paper presentation. Students present the content of 2 papers suggested by the teacher. No groups are allowed.

Teacher CV
Attach or link a max 3 pages CV for each teacher proposing the course.
https://angms.science/AAngCV.pdf

Teacher Main 5 Publications
1.**A. Ang**, Hans De Sterck, Steve Vavasis, "MGProx: A nonsmooth multigrid proximal gradient method with adaptive restriction for strongly convex optimization"
SIAM Journal on Optimization 34 (3), 2788-2820, 2024

2.**A. Ang**, J. Ma, N. Liu, K. Huang, Y. Wang,
‘‘Fast Projection onto the Capped Simplex with Applications to Sparse Regression in Bioinformatics’’ *NeurIPS21*

3.**A.M.S. Ang**, J.E. Cohen, N. Gillis, L.T.K. Hien,
‘‘Accelerating Block Coordinate Descent for Nonnegative Tensor Factorization’’,
Numerical Linear Algebra with Applications, 2021

4.**A.M.S. Ang**, J.E. Cohen, L.T.K. Hien and N. Gillis,
‘‘Extrapolated Alternating Algorithms for Approximate Canonical Polyadic Decomposition’’,
ICASSP 2020

5.V. Leplat, N. Gillis and **A.M.S. Ang**,
‘‘Blind Audio Source Separation with Minimum-Volume Beta-Divergence NMF’’,
IEEE Transaction on Signal Processing, 2020