

Course proposal

Introduction to Noncommutative Probability

Teacher

Maria Elena Griseta

Course description

The lecture is intended to introduce the basic notions, constructions, models and problems in noncommutative probability.

To this end, we first recall some preliminary notions on C^* -algebras and von Neumann algebras, and we explain the idea that led to the formulation of Algebraic Probability Space starting from the Kolmogorov triple. Subsequently, we provide definitions and properties concerning noncommutative random variables and stochastic processes.

Basic operators on the Full, Boson, Fermi, Boolean, Monotone and deformed Fock spaces are discussed, with their relations with GNS representations for involutive algebras generated by elements satisfying certain commutation relations.

We illustrate the Quantum realization of classical Brownian Motion as position and momentum operators in the Boson Fock space, and we clarify the necessity of Quantum Probability.

The crucial notion of noncommutative stochastic independence will be defined in various models, including free, monotonic, and Boolean independence. These will lead to various notions of convolutions of probability measures and noncommutative analogues of the classical Central Limit Theorem.

Course period

March-April 2025

SSD

MAT/06

Course References (optional)

- [1] Bratteli O., Robinson D. W., *Operator algebras and quantum statistical mechanics I, II*, Springer, Berlin-Heidelberg-New York (1981).
- [2] Hora A., Obata N., *Quantum probability and spectral analysis of graph, Theoretical and Mathematical Physics*. Springer Berlin (2007).
- [3] Parthasarathy K. R., *An introduction to quantum stochastic calculus*, Birkhäuser (2012).
- [4] Takesaki M., *Theory of operator algebras I*, Springer, Berlin-Heidelberg-New York, (2003).

Credits and Hours

2 credits (16 hours)

Exam Modality

Paper presentation. Students present the content of one paper suggested by the teacher. No groups are allowed.

Teacher CV

Attach or link a max 3 pages CV for each teacher proposing the course.

Teacher Main Publications

List 10 main publications in the last 15 years for each teacher.