Connections and holonomy

Teachers
Giulia Dileo – Amedeo Altavilla

Course description
Connections on vector bundles and principal bundles play a central role in differential geometry. In particular, there is a strict relation between the holonomy group of a connection and the curvature, as stated by the Ambrose-Singer theorem. For a simply connected irreducible Riemannian symmetric space G/K the holonomy group of the Levi-Civita connection coincides with the isotropy subgroup K. In the non-symmetric case the possible holonomy groups were classified by Berger.
The aim of the course is to introduce the fundamental notions of connection, torsion, curvature and holonomy, with particular attention to Riemannian manifolds. Different types of metric connections with torsion (connections of Cartan type, vectorial type and connections with totally skew-symmetric torsion) will be studied, providing explicit examples of connections adapted to special geometric structures. An introduction to calibrated geometries will be given, owing to the natural fact that Riemannian manifolds with reduced holonomy usually are endowed with one or more calibrations.

Course period
February-June 2026

SSD
MATH-02/B

Course References
[1] I. Agricola: The Srní lectures on non-integrable geometries with torsion, Arch. Math. (Brno) 42 (2006), suppl., 5-84.
[2] A. L. Besse: Einstein manifolds. Springer-Verlag, Berlin, 1987.
[3] F. R. Harvey: Spinors and Calibrations. Perspectives in Mathematics, 9. Academic Press, Inc., Boston, MA, 1990.
[4] D. D. Joyce: Compact manifolds with special holonomy. Oxford University Press, Oxford, 2000.
[5] S. Kobayashi, K. Nomizu, Foundations of differential geometry. Vol. I-II. Wiley-Interscience, New York · London, 1996.

Credits and Hours
3 credits of lectures for a total of 24 hours divided in:

- 16 hours - Giulia Dileo
- 8 hours - Amedeo Altavilla

Exam Modality
Seminar on a topic or a paper related to the contents of the course.

Teachers CV

https://www.dm.uniba.it/it/members/dileo/homepage/cv_dileo.pdf

https://www.dm.uniba.it/it/members/altavilla/cv_italiano-senza-dati-sensibili.pdf

Teachers Main Publications

Giulia Dileo:

1. D. Di Pinto, G. Dileo, *Anti-quasi-Sasakian manifolds,* Ann. Glob. Anal. Geom. 64 (2023), Article No. 5, 35 pp.
2. I. Agricola, G. Dileo, L. Stecker, *Curvature properties of 3-(α,δ)-Sasaki manifolds,* Ann. Mat. Pura Appl. 202 (2023), no. 5, 2007-2033.
3. A. Andrada, G. Dileo, *Odd-dimensional counterparts of abelian complex and hypercomplex structures*, Math. Nachr. 296 (2023), no. 2, 470-508.
4. I. Agricola, G. Dileo, L. Stecker, *Homogeneous non-degenerate 3-(α,δ)-Sasaki manifolds and submersions over quaternionic Kähler spaces*, Ann. Glob. Anal. Geom. 60 (2021), no. 1, 111-141.
5. I. Agricola, G. Dileo, *Generalizations of 3-Sasakian manifolds and skew torsion*, Adv. Geom. 20 (2020), no. 3, 331-374.
6. G. Dileo, A. Lotta, *A note on Riemannian connections with skew torsion and the de Rham splitting*, Manuscripta Math. 156 (2018), no. 3-4, 299-302.
7. A. De Nicola, G. Dileo, I. Yudin, *On nearly Sasakian and nearly cosymplectic manifolds*, Ann. Mat. Pura Appl. 197 (2018), 127-138.
8. B. Cappelletti Montano, G. Dileo, *Nearly Sasakian geometry and SU(2)-structures*, Ann. Mat. Pura Appl. 195 (2016), 897-922.
9. G. Dileo, A. Lotta, *Riemannian almost CR manifolds with torsion*, Illinois J. Math. 58 (2014), no. 3, 807-846.
10. G. Dileo, A. Lotta, *Levi-parallel contact Riemannian manifolds*, Math. Z. 274 (2013), no. 3-4, 701-717.

Amedeo Altavilla:

1. A. Altavilla, E. Ballico, M. C. Brambilla, Twistor fibers in hypersurfaces of the flag threefold. New York Journal of Mathematics, 2023, 29, pp. 1117–1148.
2. A. Altavilla, C. de Fabritiis, $*$-Logarithm for Slice Regular Functions, Atti Accad. Naz. Lincei Cl. Sci. Fis. Mat. Natur. 34 (2023), no. 2, pp. 491–529.
3. A. Altavilla, E. Ballico, M. C. Brambilla, S. Salamon, Twistor geometry of the Flag manifold. Math. Z. 303, 24 (2023).
4. A. Altavilla, E. Ballico, M. C. Brambilla, Surfaces in the Flag Threefold Containing Smooth Conics and Twistor Fibers. Mediterr. J. Math. 19, 281 (2022).

5. A. Altavilla, S. Mongodi, Slice Regular Functions as Covering Maps and Global $*$-Roots. J Geom Anal 32, 207 (2022).
6. Altavilla, A. Spherical Coefficients of Slice Regular Functions. Results Math 76, 178 (2021).
7. Altavilla, A., Arosio, L., Guerini, L. Canonical Models on Strongly Convex Domains via the Squeezing Function. J Geom Anal (2020).
8. A. Altavilla, H. De Bie, M. Wutzig, Implementing zonal harmonics with the Fueter principle, Journal of Mathematical Analysis and Applications Volume 495, Issue 2, 15 March 2021, Article number 124764.
9. A. Altavilla, C. de Fabritiis. Applications of the Sylvester operator in the space of slice semi-regular functions, Concr. Oper., (2020) 7(1), 1-12.
10. A. Altavilla, E. Ballico, Algebraic surfaces with infinitely many twistor lines, B. Aust. Math. Soc. (2020) 101(1), 61-70.